The Assessment of Visual Thinking of the Concept of Mat1hematics

Ummu Sholihah[1],Toto Nusantara[2], Cholis Sa'dijah[3], Hery Susanto[4]
Universitas Negeri Malang, Indonesia
ummu2280@yahoo.com[1], cholis.sadijah.fmipa@um.ac.id[3]

Abstract: The learning of calculus movement emphasizes the use of multiple representations in the presentation of concepts, that concepts should be represented numerically, algebraically, graphically and verbally wherever possible, so that students understand connections between different representations and develop deeper and more robust understanding of the concepts. Visualization is a critical aspect of mathematical thinking, understanding and reasoning. Therefore, it is necessary to consider assessment instrument to support the process of visual thinking. The purpose of this study is to describing assessment characteristics to support student's visual thinking of mathematics, especially the concept of integral. Data sources of this study are student teachers at the fifth semester. The data are in the form of test and interview result. The finding of this study show that there are three characteristics of assessment that can support visual thinking, which are (1) non-visual (NV) means that the assessment tool should make the student inability solve the problem using algebra and geometry, inability to represent and interpret the problem graphically (2) local-visual (LV) means that the assessment tool should be able to measure the ability of students to learn algebra and geometry as an alternative language, capable of drawing and using diagrams in solving the problem (3) global-visual (GV) means an assessment instrument must contain the student is able to understand the interrelationships between representations of concepts learned by the students.

Keywords: assessment, visual thinking, mathematics.

Visualization has been an area of interest for a number of researchers concerned with mathematics education. Many researchers emphasize the importance of visualization and visual reasoning for learning mathematics, and visualization is a fundamental aspect in understanding students' construction of mathematical concepts (Chih-Hsien Huang, 2013: 111)

Some studies show the importance of visual thinking. (Hartono, 2010: 1) tells us that Plato's analogy of memory as a block of wax and shaping perceptions of mold on it. Aristotle explained that the mold is in the form of 'image' and the language is the representation. Nemirovsky & Noblemany (1997:1) researchers emphasize the importance of visualization and visual reasoning for learning mathematics. In other words, the researchers suggest the importance of visualization and visual reasoning to study mathematics. Arcavi (2003) defines the visual thinking as the ability, the process and the creation, interpretation, use and the idea of images, pictures and diagrams in the mind, on paper or using technological tools, with the aim of describing and communicating information and ideas, develop previous ideas and increase understanding. Visual thinking is also as the ability to represent, transform, generalize, communicate, document and reflect the objects or objects into visual information. Furthermore, the visual thinking as the ability to change the information of all types into images, graphics or other forms of information that can help communicates.

Visual thinking helps assure correctness proof of a theorem. (Tall, 1991: 2) describes it as follows:

In mathematical research proof is but the last stage of the process. Before there can be proof, there must be an idea of what theorems are worth proving, or what theorems *might be true. This exploratory stage of mathematical thinking benefits from building up an overall picture of relationships and such a picture can benefits from visualization.*

Visualization is a critical aspect of mathematical thinking, understanding, and reasoning. Researchers argue that visual thinking is an alternative and powerful resource for students to do mathematics; it is different from linguistic, logic-propositional thinking and manipulation of symbols. A growing body of research supports the assertion that understanding of mathematics is strongly related to the ability to use visual and analytic thinking. Researchers contend that in order for students to construct a rich understanding of mathematical concepts, both visual and analytic reasoning must be present and integrated (Aspinwall, 2002: 434-440).

According to Duval (Duval, 2006: 103-131) visualization can be produced in any register of representation as it refers to processes linked to the visual perception and then to vision. Zimmerman and Cunningham (1991: 127-138) contended that the use of the term “visualization” concerned a concept or problem involving visualizing. Nemirovsky and Noble (1997: 595-610) defined visualization is a tool that penetrated or travelled back and forth between external representations and learners’ mental perceptions. Goldin (1998: 137-165) and Hitt (1998: 1-7) both emphasized the relationships among representation, mathematical visualization, and conceptual understanding. Dreyfus (1991: 33-48) contended that what students “see” in a representation would be linked to their conceptual structure, and further proposed that visualization should be regarded as a learning tool. Noss, Healy, and Hoyles (1997: 203-233) described mathematical thinking as being characterized by the ability to move freely between the visual and the symbolic, the formal and the informal, the analytical and the perceptual, and the rigorous and the intuitive.

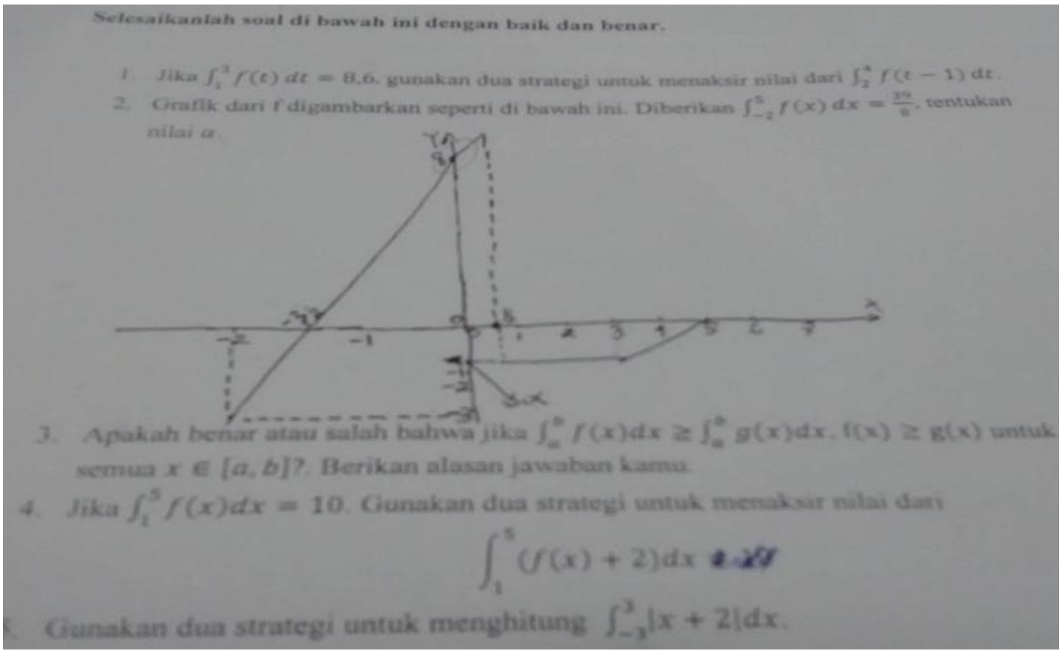
Visualization involves both external and internal representations (or images), and thus following Presmeg (2006: 205-235) we define visualization as processes involved in constructing and transforming both visual images and all of the representations of a spatial nature that may be used in drawing figures or constructing or manipulating them with pencil and paper. This definition emphasizes that in mathematical thinking and problem solving, an appropriate graph can be drawn to represent the mathematical concept or question, and that the graph can be used to understand a concept or as a problem-solving tool. In this study, we investigated the visual images that students used to resolve specific problems and how they managed given visualizations.

This study stands apart from other research on learning calculus, because it not only extends the understanding of students’ difficulties and strengths associated with visualization, but also identifies the types of visual image they utilized while solve integral problems.

METHODOLOGY

Participants and Instruments

The 20fifth semester teachers of mathematics students who participated in this study were enrolled at IAIN Tulungagung. The data are in the form of test and interview result. The questionnaire comprised five problems indefinite integral (Figure 1), some of which were referenced from other studies.



Selesaikanlah soal di bawah ini dengan baik dan benar.

1. Jika $\int_1^3 f(t)\,dt = 8{,}6$. gunakan dua strategi untuk menaksir nilai dari $\int_2^4 f(t-1)\,dt$.
2. Grafik dari f digambarkan seperti di bawah ini. Diberikan $\int_{-2}^{5} f(x)\,dx = \frac{39}{8}$, tentukan nilai a.
3. Apakah benar atau salah bahwa jika $\int_a^b f(x)dx \geq \int_a^b g(x)dx$, $f(x) \geq g(x)$ untuk semua $x \in [a, b]$?. Berikan alasan jawaban kamu.
4. Jika $\int_1^5 f(x)dx = 10$. Gunakan dua strategi untuk menaksir nilai dari
$$\int_1^5 (f(x) + 2)dx$$
5. Gunakan dua strategi untuk menghitung $\int_{-3}^{3} |x+2|dx$.

Figure 1 Problems in Definite Integral

The results of the questionnaire necessitated further investigation into the visual thinking of the students. The clinical interviews were carried out after the answers to the problems had been analyzed (Goldin, 2000: 517-545) . Each interview lasted about 20-30 minutes and was video-and audio-taped. In order to prepare the script for the interview, the author analyzed the written answers focused on to how the students seemed to use and coordinate the different mathematical representations needed. During each interview the students were asked to think aloud, while they were solving the tasks so that the author could describe their responses and strategies, as well as make inferences about their mental processes and images.

The analysis results indicated that the visual thinking distribution of 30 students could be categorized into five competencies and three levels. In the non-visual (NV) level, one tends to focus on a single visual image, overlooking other representations of a similar nature. In the local visual (LV) level, one can perceive and confirm the relationships among various visual images; however, these items may still appear independent of each other. In the global-visual (GV) level, one can use the relationships to construct a consistent structure based on the relationships among various visual images.

RESULTS AND DISCUSSION

According to the data analysis, the author identified five competencies of visual thinking relating to the concept of definite integral and, then, classified the visual thinking of the students into three levels. Because students' visual thinking of the definite integral could be reasonably understood regarding the three levels, the author evaluated the responses to the task interviews, searching for evidence of the NV, LV and GV levels.

Visual Thinking of the Concept of Definite Integral in the NV Level

According to the data analysis the author categorized nine students into this group. One of the visual thinking characteristics shared by these students was that they could not recognize the relationship between the area and integral, they could not understand algebra and geometry as alternative languages, they could not extract specific information from diagrams, they could not represent and interpret problem (or concept) graphically, they could not draw and use diagrams as an aid in problem-solving, they could not understand mathematical transformations visually. These students could only process representations within a representation system, and the representations used were influenced by the representation format employed for problems. Additionally, they preferred solving problems using symbolic representations.

Consider the following excerpt from the interview conducted with Shafiq, who has a collection of rules that enable him to integrate fundamental functions, such as the integrals in Tasks 1 and 2. The students could solve a number of problems by simply applying rules that had been memorized and in some cases, incorrectly remembered. However, he could not solve the problems using graphical representations. As the figure below show the results of the completion of the number 1.

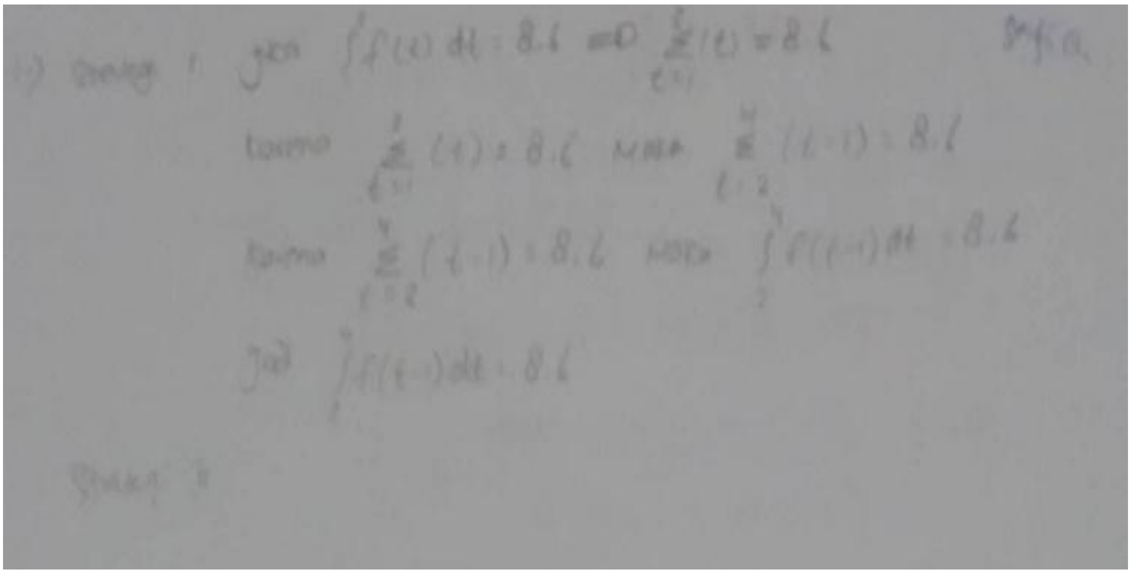


Figure 2 Safiq's Presentation for Task 1

Visual Thinking of the Concept of Definite Integral in the LV Level

The next level of visual thinking of the concept of definite integral is the local-visual level. According to the data analysis the author categorized seven students into this group. One of the visual thinking characteristics shared by these students was that they could recognize the relationship between the area and integral, they could demonstrated the competency completely to understand algebra and geometry as alternative languages, they could demonstrated the competency completely to extract specific information from diagrams, they could not demonstrated the competency fully to represent and interpret problem(or concept)graphically, they could not demonstrated the competency fully to draw and use diagrams as an aid in problem-solving, they could demonstrated the competency completely to understand mathematical transformations visually. These students understood the relationships between representation systems and could change or transfer the representations in some of the representation systems. However, these students had difficulty coordinating these relationships. For example, Miftah was one of the students in this group. She could use correct symbolic representations to perform mathematical thinking and could manipulate the area using graphical representations according to the changes in integral symbols in Tasks 1 and 4. As shown below

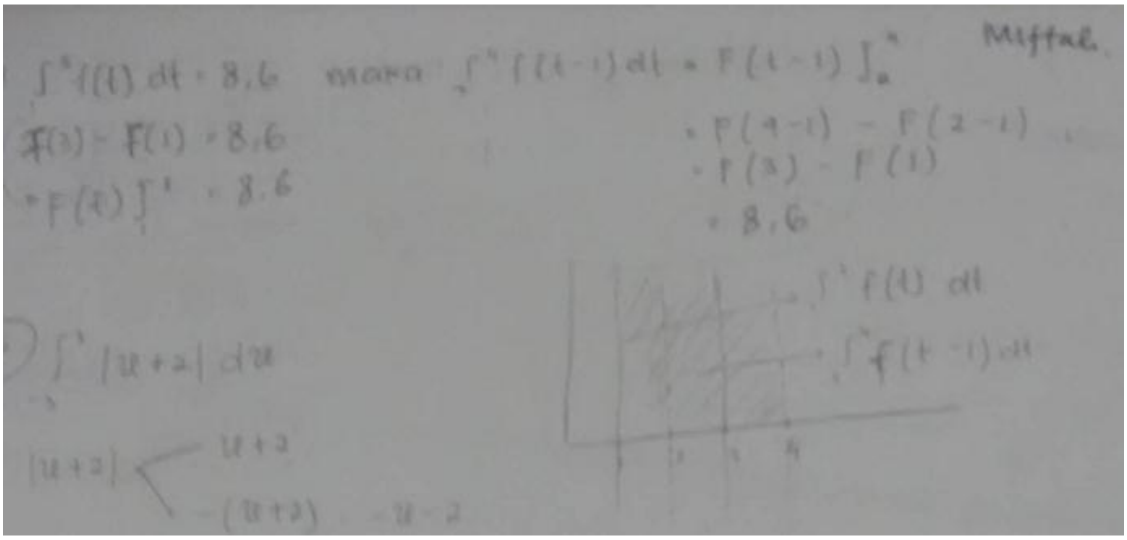


Figure 3. Miftah's Presentation for Task 1

Visual Thinking of the Concept of Definite Integral in the GV Level

Four students were categorized into this group. These students could recognize the relationships among representation systems and convert representations between representation systems. In Tasks 1, 4 and 5, Yanuar used the correct symbolic representations to perform mathematical thinking. She also manipulated the area using graphical representations according to the changes in integral symbols. They could demonstrated the competency completely to understand algebra and geometry as alternative languages, they could demonstrated the competency completely to extract specific information from diagrams, they could demonstrated the competency completely to represent and interpret problem(or concept)graphically, they could demonstrated the competency completely to draw and use diagrams as an aid in problem-solving, they could demonstrated the competency completely to understand mathematical transformations visually. For example, Yanuar was one of the students in this group. She could use correct symbolic representations to perform mathematical thinking and could manipulate the area using graphical representations according to the changes in integral symbols. As shown below

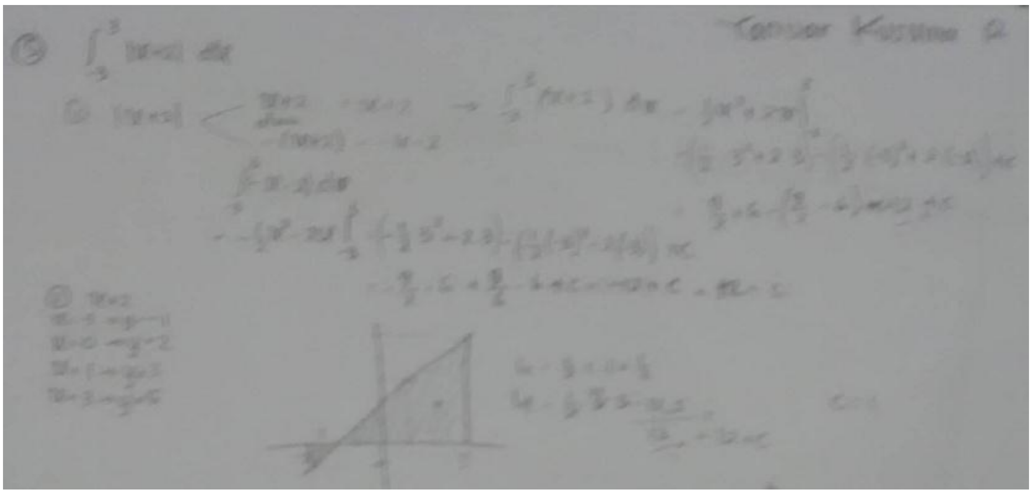


Figure 4 Miftah's Presentation for Task 4

CONCLUSION

Visual thinking involves the capacity to make connections between both mathematical objects and concepts and mathematics and the physical world. According the data analysis results show that the main obstacles preventing students from freely shifting within the representation system for the concept of definite integrals were that they did not have the ability involves visualizing the abstracted relationships and non-figural information into visual representations and imagery. The development of visualization ability, which may influence the relationship between graphical representations and the other representations, increases the performance of solving definite integral problems.

The students (1) non-visual (NV) means that the assessment tool should make the student inability solve the problem using algebra and geometry, inability to represent and interpret the problem graphically (2) local-visual (LV) means that the assessment tool should be able to measure the ability of students to learn algebra and geometry as an alternative language, capable of drawing and using diagrams in solving the problem (3) global-visual (GV) means an assessment instrument must contain the student is able to understand the interrelationships between representations of concepts learned by the students.

REFERENCES

Arcavi, A. 2003. The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52, 215-241.

Dreyfus, T. 1991. On the status of visual reasoning in mathematics and mathematics education. In *Proceedings of the Fifteenth Annual Meeting of the International Group for the Psychology of Mathematics Education,* Assisi, Italy, 33-48.

Duval, R. 2006. A cognitive analysis of problems of comprehension in a learning of mathematics." *Educational Studies in Mathematics*, 61(1). 103-131.

Goldin, G. 1998. Representational systems, learning, and problem solving in mathematics. *Journal of mathematical behavior,* 17 (2). 137-165.

Goldin, G. 2000. *Handbook of Research Design in Mathematics and Science Education*, Erlbaum, Mahwah, New Jersey, 517- 545

Hartono. 2010. *Mental Imagery: tinjauan dari segi filsafat, Ilmu-ilmu Kognitif dan Neurologis.* Surabaya: UNESA University Press.

Hitt, F. 1998. "Working Group on Representations and MathematicsVisualization. In *Proceedings of the Twentieth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, USA. Columbus, 17.

Huang, Chih-Hsien. 2013. Engineering Students' Visual Thinking Of The Concept Of Definite Integral. Global Journal of Engineering Education Volume 15, Number 2,: 111-117

Nemirovsky & Noblemany . 1997. *On Mathematical Visualization and the Place Where we Live.* Educational Studies in Mathematics 33: 99-131. Kluwer Academic Publisher, Printed in The Netherlands.

Nemirovsky, R., and Noble, T. 1997. "On Mathematical Visualisation and the Place Where We Live. *Educational Studies in Mathematics* 33(2). 595-610.

Noss, R., Healy, L. and Hoyles, C. 1997. The Construction of Mathematical Meanings: Connecting the Visual with the Symbolic." *Educational Studies in Mathematics,* 33(2). 203-33.

Presmeg, N.C. 2006. *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future*. Sense Publishers, Rotterdam 205-235

Tall. 1991. *Intuition and Rigour: The Role of Visualization in The Calculus. Visualization in Mathematics.* Notes No. 19, 105-119.